A review about trajectory control techniques for interior permanent magnet synchronous motors

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Resumen / Abstract

El motor sincrónico de imán permanente posee características especiales para operar a velocidad variable que lo distinguen de los otros tipos de motores de corriente alterna (CA). Debido a la configuración especial del rotor, su salida y su relativamente alta inductancia, este motor es adecuado para operar a flujo debilitado en un amplio diapason, ofreciendo además una mayor relación momento/volumen. Existen muchas técnicas reportadas en la literatura para el seguimiento de la corriente y el momento. La técnica de control momento/corriente es una de ellas, y puede ser a su vez, subdividida en directa, instantánea e indirecta. Este trabajo hace un análisis de dos de ellas: La de máximo momento por ampare con control PWM y la del control instantáneo del momento. Se discuten también las múltiples ventajas de utilizar estas técnicas.

Palabras clave: Motor sincrónico de imán permanente, control instantáneo del momento, flujo debilitado

The interior permanent magnet synchronous motor (IPMSM) possesses special features for adjustable speed operation that distinguish it from other type of AC machines. Because of its special rotor configuration, pole saliency and relatively large armature inductances, the IPM is also more suitable for an extended speed range operation by the flux weakening control, in addition to the higher torque/volume it can offer [1]. There are many techniques, for controlling the actual torque or current to follow their references, reported in the literature for this type of motor. The torque/current control technique is one of them and can be classified into direct, instantaneous and indirect torque controls. This work reviewed two of them, i.e. maximum torque-per-ampere (MTPA) techniques in PWM current control and Instantaneous torque control for Interior Permanent Magnet Synchronous Motor. The multiple advantages of the use of this type of techniques are discussed too.

Key words: Permanent magnet synchronous motor (PMSA), instantaneous torque control (ITC), flux weakening

INTRODUCTION

The interior permanent magnet synchronous motor (IPMSM) possesses special features for adjustable speed operation that distinguish it from other type of AC machines. Because of its special rotor configuration, pole saliency and relatively large armature inductances, the IPM is also more suitable for an extended speed range operation by the flux weakening control, in addition to the higher torque/volume it can offer. There are many techniques, for controlling the actual torque or current to follow their references, reported in the literature for this type of motor. The torque/current control technique is one of them and can be classified into direct, instantaneous and indirect torque controls. For the latter one, it is mainly carried out by the current control. Both, torque and current control can be performed in the stator/stationary reference frame or in rotor/stator flux reference frame. For first reference frame named, the most commonly used controllers
are hysteresis current controller and PWM current controller.

Traditionally, the PWM current control in the stationary reference frame is simple and easy to implement; however, its transient and steady state performances, are the functions of load, machine impedance and operating frequency. With the appearance of high-speed microprocessors, which provides fast calculation and A/D conversion for current measurement, the PWM current control in the synchronous reference frame has attracted more and more attention in the last decade. The back emf disturbance and cross-coupling effect between controllers are eliminated in the steady state in this reference frame and the dynamic response of the synchronous current controller is generally much faster that the one in stationary reference frame in terms of response time and accuracy.

The speed control loop not only performs the speed control but also generates the reference torque/current for its inner control loop according to the selected control algorithm, which is called trajectory control. Due to the pole saliency of the IPM synchronous motor, trajectory control techniques, which can offer the optimized constant torque and field weakening operation for IPM motor, are very important and necessary.

This work reviewed two of them, i.e. maximum torque-per-ampere (MTPA) techniques in PWM current control and Instantaneous torque control for Interior Permanent Magnet Synchronous Motor.

**TRAJECTORY CONTROL TECHNIQUES**

There are many trajectory control algorithms for produce the optimal references for the inner torque/current and field control loops. It is mainly based on the current control in the rotor flux reference frame, and are classified into four categories, namely:

- Id=0 control, where the reluctance torque is not fully utilized with such a control and the efficiency of the drive is the rather low.
- Maximum torque-per-ampere (MTPA) control, the higher torque can be produced in addition to improving the efficiency and the power factor when compared to the conventional id=0 control.
- Unity power factor (UPF) control, not suitable for high-performance variable-speed drives but only suitable for constant speeds applications.
- Field weakening (FW) control, there are three operation modes, if the operation mode is selected properly, not only is smooth transition between constant torque and field weakening operations are obtained, but also the current and voltage limitations are satisfied.

There are some other techniques of trajectory control, such as:

- Input power minimization control.
- Constant flux lineage control.

**Maximum Torque-per-Ampere control (MTPA)**

Jahns et al. was the first person that proposed the Maximum Torque-per-ampere control technique, for achieving high efficiency operation of IPM synchronous motors. It was shown that MTPA trajectory is a tangent to the q-axis at origin (rotor flux reference frame axes) and asymptote to 45 degrades trajectories. The excitation torque is dominant when torque is low. If the torque increases, the reluctance torque, which is proportional to the square of the stator current, is increasingly dominant. Morimoto et al. following Jahns’ study, revealed further that with MTPA control, higher torque can be produced when compared the conventional control with id = 0, in addition to improving the efficiency and the power factor.

The voltage and torque equations of an IPM synchronous motor may be expressed in the dq-axis reference frame, which rotates in synchronism with the rotor and its d-axis is fixed on the rotor flux.

\[
\begin{bmatrix}
  v_d \\
  v_q
\end{bmatrix} = \begin{bmatrix}
  r + pL_d & -\omega L_q & i_d \\
  \omega L_d & r + pL_q & i_q
\end{bmatrix} + \begin{bmatrix}
  0 \\
  \omega \varphi
\end{bmatrix}
\]

\[
T = \frac{3}{2} P\varphi i_q + \frac{3}{2} P(L_d - L_q)i_d i_q
\]  

Where:

- \( i_d \) and \( i_q \) are the dq-axis components of the armature current and voltage, respectively.
- \( r \) is the armature resistance.
- \( \varphi \), \( L_d \) and \( L_q \) are the armature (or stator) back emf constant and inductances.
- \( w \) is the electrical angular velocity.
- \( P \) is the number of poles.
- \( p \) is \( d/dt \).

The operation of the motor torque in equation (2) is the excitation torque \( T_e \) and the second term is the reluctance torque \( T_r \). Both components of the motor torque should be controlled properly for high torque and high efficiency.
The implementation of this control is based on the real-time calculation of current reference. The motor can produce maximum available torque by the maximum torque-per-ampere control below base speed and can also operate in the constant power region by the flux weakening within the maximum voltage limitation above the base speed.

To obtain fast transient response and high torque, the current phase angle must be controlled to develop the maximum torque. It has been demonstrated analytically that the dq-axis of the armature current for maximum torque-per-ampere is:

\[
\frac{3}{2}P\varphi_{d}i_{d} + \frac{3}{2}P\varphi_{q}(L_{q} - L_{d})(i_{d}^{2} - i_{q}^{2}) + 0 \quad \ldots(3)
\]

\[
i_{d}^* = \frac{\varphi_{d}}{2(L_{q} - L_{d})} - \frac{\varphi_{d}^{2}}{4(L_{q} - L_{d})^{2}} + i_{q}^{2} \quad \ldots(4)
\]

Equation (4) implies that the maximum torque-per-ampere is obtained \(i_{d}, i_{q}\) is determinate \(b, (4)\) for any \(i_{d}^{*}\). \(i_{q}\) should be noted that the torque is not directly proportional to \(i_{d}^{*}\). This is the reason that the torque control via current control is called indirect torque control in some literatures.\(^{13}\)

**Current and voltage trajectory control**

When the IPM synchronous motor is fed from an inverter, the armature current and voltage may not exceed the inverter current and dc-link voltage ratings. These can be expressed as:

\[
I_{s} = \sqrt{i_{d}^{2} + i_{q}^{2}} \leq I_{sm} \quad \ldots(5)
\]

\[
V_{s} = \sqrt{V_{d}^{2} + V_{q}^{2}} \leq V_{sm} \quad \ldots(6)
\]

Where \(I_{sm}\) and \(V_{sm}\) are the available maximum current and voltage of the motor/inverter, respectively.

Substituting equation (1) into equation (6) yields:

\[
V_{s} = \sqrt{(\omega L_{q} i_{q}^{2} + \omega L_{d} i_{d} + \omega \varphi_{d})^{2}} \leq V_{sm} \quad \ldots(7)
\]

Simplified

\[
(L_{q} i_{q}^{2} + L_{d} i_{d} + \varphi_{d})^{2} \leq \left(\frac{V_{sm}}{\omega}\right) \quad \ldots(8)
\]

Solving \(i_{d}\) from equation (3) and equation (5), and substituting in equation (2), the maximum torque-per-ampere, current and voltage limits trajectories can be rewritten to related torque and \(i_{d}\) as:

\[
T^* = \frac{3}{2}P\sqrt{i_{d}^{2} - \frac{\varphi_{d}}{L_{q} - L_{d}} + i_{d}^{*}(\varphi_{d} + (L_{d} - L_{q})i_{d}^{*})} \quad \ldots(9)
\]

\[
T^* \leq \frac{3}{2}P\sqrt{i_{d}^{2} - \frac{\varphi_{d}}{L_{q} - L_{d}} + i_{d}^{*}(\varphi_{d} + (L_{d} - L_{q})i_{d}^{*})} \quad \ldots(10)
\]

**PWM current control**

The general block diagram used more frequently for this control is shown in figure 1, and separate \(i_{d}\) and \(i_{q}\) current control are performed in the rotor flux reference frame. Therefore, only sinusoidal phase currents can be produced in steady state if the reference currents are constants. Two line currents are measured by isolated current transducers and the third line current is calculated from the current constraint for Y-connected stator windings. An absolute encoder measures the rotor position.

The trajectories of \(i_{d}\) and \(i_{q}\) satisfying the MTPA characteristics and current and voltage limit constraints are represented by equations (3), (5) and (8), respectively. The current limit trajectory according to the equation (5) is a circle centered at origin and with the radius of \(I_{sm}\). The voltage limit trajectory represented by the equation (8) is an ellipse.

![Block diagram of a current controlled IPMSM.](image)
The MTPA and current limit trajectories are independent of the rotor speed and are only determined by the motor parameters and inverter current rating. However, the voltage limit trajectory varies with the change in rotor speed. When the rotor speed increases infinitely, the voltage limit ellipse becomes a point on the d-axis. If this point is inside the curve of current limit, the motor has an infinite maximum speed, if not, has finite maximum speed.

The intersection of MTPA current limit trajectories is the operation point at which the motor has the rated current and voltage, and will produce the rated torque at the base speed.

According to the rotor speed, the motor operation is divided in three sections.

**Operation below the base speed**

If the stator current vector is controlled according to the maximum torque-per-ampere trajectory and satisfying the current limit, it must satisfy the voltage limit since the current vector is inside the voltage limit ellipse. The d- and q- axis currents, \( i_d \) and \( i_q \), with which the maximum torque is produce, are determined when \( I_s = I_{sm} \) by:

\[
i_d = \frac{\varphi_l L_d}{4(L_q - L_d)} - \sqrt{\frac{\varphi_l^2}{16(L_q - L_d)^2} + \frac{i_{sm}^2}{2}} \quad \ldots (12)
\]

\[
i_q = \sqrt{i_{sm}^2 - i_d^2} \quad \ldots (13)
\]

**Operation between the base and crossover speed**

The determination of the control mode is based on the calculated id from both equations (4) and (8). If the calculated id from equation (4) is smaller than the one calculated from (8), the current vector is controlled to the MTPA trajectory for constant torque operation. Otherwise, voltage limit trajectory is used to control the current vector for field weakening operation.

**Operation above the crossover speed**

In this case, the flux linkage along the rotor d axis has to be reduced with the MTPA control. The voltage limitation is no longer satisfied when the rotor speed is above the crossover speed. The stator current vector is therefore controlled according the voltage limit trajectory instead of MTPA trajectory.

The limit values \( i_{q} \) and \( i_{q} \) of the outer control loop for such a field weakening operation are therefore determined by these two equations with \( I_s = I_{sm} \), which is inversely proportional to the rotor speed.

\[
i_d = \frac{\varphi_l L_d}{a} - \frac{1}{a} \sqrt{\varphi_l^2 - \frac{i_d^2}{2} - ab} \quad \ldots (14)
\]

\[
i_q = \sqrt{i_{sm}^2 - i_d^2} \quad \ldots (15)
\]

where, \( a = L_d^2 - L_q^2 \) and \( b = \varphi_l^2 + L_d^2 L_q^2 - \left( \frac{V_{sm}}{a} \right)^2 \)

The simulations results from a number of authors with this control shown that the IPM synchronous motor can operate in both constant torque and field weakening regions with very fast dynamics.

**INSTANTANEOUS TORQUE CONTROL**

To implement the trajectory control in the instantaneous torque control scheme, usually, the outer speed controller produces torque reference and the d-axis current reference is determined for constant torque or field weakening. See figure 2.

The control trajectories are transformed into torque-id plane. The maximum torque the motor can develop and the maximum speed of the motor for a given load are clearly shown in this plane. However, the expressions of \( i_d \) in terms of torque for the maximum torque-per-ampere control and the voltage and current limit trajectories are quite complicated. To overcome this complexity, a Newton-Raphson approximation can be used to calculate \( i_d \) from torque in real time.

In the constant torque region the voltage limit is always satisfied and the torque and \( i_d \) are controlled according to equations (9) and (10). Above the crossover speed, at which the back emf generated by the magnet only is equal to Vsm, the torque and \( i_d \) are controlled according to equations (10) and (11) for field weakening operation. In the speed region from base speed to the crossover speed, the reference for \( i_d \) is calculated from equation (9) for a given reference torque. If the resultant torque is smaller than the reference torque, equation (11) is selected to calculate reference \( i_d \) for flux weakening operation. If not, equation (9) is still used for constant torque operation.

**COMPARISON OF CONTROLS**

Due to its simplicity and constant switching frequency properties, the PWM current control structure is widely used for torque control of PM synchronous motor drive.
However, the speed is not very smooth due to the cogging and ripple torques. This is the problem of interior PM synchronous motor drives under this type of control. The torque and field can not be controlled separately because both $i_d$ and $i_q$ currents contribute to the torque. As a result, the relationship between the output of the speed controller and the motor torque will not be linear.

Instantaneous torque control gives solution to the problems planted before. The starting point of the development of this drive is to separate the control of the torque and the field and to obtain the linear relationship between the outputs of the speed control and the torque. A torque controller is used to replace the $i_d$ current controller and id current is also controlled indirectly. However, the expressions of $i_d$ in terms of torque for the maximum torque per ampere control and the voltage and current limit trajectories are quite complicated and are represented by forth order polynomial equations, but in the actuality this is not a problem, because, there are many methods of approximation developed. The reference torque can be easily modified accordingly to obtain smooth speed response.

**REFERENCES**


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**CONCLUSIONS**

The main trajectory techniques for the control of interior permanent magnet synchronous motors have been described. The multiple advantages of the use of this type of techniques are discussed too. It has been possible to appreciate in literature the importance of the control methods. Recently, these techniques has been studied to applied for high-performance applications of the PM synchronous motor such as the high-precision torque and speed controls of the machine tools and industrial robots.


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